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大型容器活塞底板的径向位移及协调条件理论分析*

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摘 要 为考虑活塞桁架与底板之间的剪切作用，本文将容器活塞底板视为部份环形板处理，设活塞桁架作用在底板上的剪力按幂级数规律分布，将问题归结为部份环板的合边值问题。文中导出了这一弹性力学平面问题的解答，给出了底板径向位移的一般表达式，并对底板与桁架下弦各节点以及中心筒壳处的位移协调条件进行了分析。

关键词 部份环板，平面问题，幂级数

1 问题的力学模型

图1为某大型容器活塞构架的示意图。由于其结构型式复杂，处理不易，工程界一直难于作出正确可行的力学分析。如仅考虑桁架和中心筒壳的共同作用，不计底板的影响，则得出与实际情况有较大误差的结果。文〔3〕在文〔1〕，〔2〕基础上考虑了底板横向弯曲作用，结果有了较大的改进，但未计入底板与构架之间的剪切作用。事实上，由于底板在板的平面内刚度很大，在考虑荷载作用下，整个结构的径向变形和受力时，底板必然起相当大的作用。合理的计算模型除了应考虑底板与桁架共同工作时的横向弯曲作用，还必须考虑二者之间的剪切效应和径向位移协调条件。因此对活塞底板的径向位移进行正确的分析，是整个容器活塞结构准确分析的关键问题之一。

考虑到结构的对称性，桁架所在平面均为底板的对称面，我们取底板中两品桁架之间的部份进行研究，可以得出图2的计算模型。由于一般情况下容器的边数$h\geqslant 20$，故每边张角均较小，$2\alpha\leqslant\frac{\pi}{10}$，我们可以把研究对象作为部份圆环板的平面问题处理，外边界简支，内边界与中心筒壳联接，视为弹性约束。由于$\frac{r_0}{R}\ll 1$，一般工程问题中$\frac{r_0}{R}$约为0.02，我们可以认

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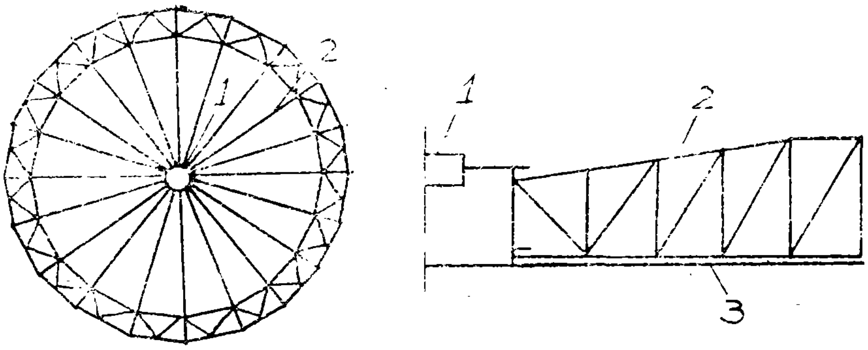

图 1 大型容器结构架示意图

1——中心圆筒 2——桁架 3——底板

为底板给中心筒壳的作用力在内边界均匀分布，内边界处的径向位移同样也为均匀分布，这种处理可大为简化计算，并可满足工程的要求。

设桁架作用在底板上的剪力按幂级数规律分布，

$$q(r)=\sum_{m=0}^{n} q_m r^m$$

其中$q_m(m=0, 1\cdots, n)$为待定常数，取级数的项数为桁架下弦节点数$n+1$。

因此问题归结为处理图2部份环板的混合边值问题。求解该弹性力学平面问题，可以得出底板在桁架各节点处及与中心筒壳联接处的径向位移，根据上述部位底板与桁架和筒壳的位移协调条件，可得$n+1$个径向位移协调方程，求解方程组，可以确定待定系数$q_0, q_1\cdots, q_n$，从而得出问题的解答。

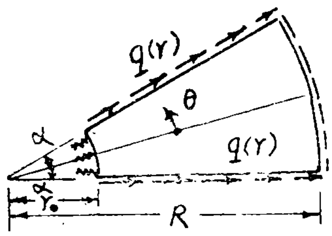

图 2 部分环形圆板示意图

2 部份环板混合边值问题的解答

由文[4]并考虑到问题的对称性，应力函数取为

$$\Phi=(E_1\ln\rho+E_2\rho^2+E_3\rho^2\ln\rho)+\left(G_1\frac{1}{\rho}+G_2\rho\ln\rho+G_3\rho^3\right)\cos\theta$$

$$+\sum_{m=2}^{n}[A_m\rho^{-m}+B_m\rho^{2-m}+C_m\rho^m+D_m\rho^{2+m}]\cos m\theta$$

$$+(K_1\rho+K_2\rho\ln\rho)\theta\sin\theta$$

式中 $\rho=\frac{r}{R}$，E_1、E_2、E_3、G_1、G_2、G_3、K_1、K_2、A_m、B_m、C_m、D_m 均为待定系数。设径向位移及环向位移的$1/R$倍为u_ρ、u_θ。则边界条件为

$$\theta=\alpha \qquad \tau_{\theta\rho}=\sum_{m=0}^{n} q_m\rho^m \qquad (1)_1$$

$$u_\theta=0 \qquad (1)_2$$

$$\rho=1 \qquad \sigma_\rho=0 \qquad (1)_3$$

应力分量式为

$$\tau_{\rho\theta}=\frac{1}{\rho^2}\frac{\partial\Phi}{\partial\theta}-\frac{1}{\rho}\frac{\partial^2\Phi}{\partial\rho\partial\theta}$$

$$=\left[-2G_1\frac{1}{\rho^2}+(G_2-K_2)\frac{1}{\rho}+2G_3\rho\right]\sin\theta-\frac{K_2}{\rho}\theta\cos\theta$$

$$+\sum_{m=2}^{n}[-m(1+m)A_m\rho^{-m-2}+m(1-m)B_m\rho^{-m}+m(m-1)C_m\rho^{m-2}$$

$$+m(m+1)D_m\rho^m]\sin m\theta \qquad (2)$$

$$\sigma_\theta=\frac{\partial^2\Phi}{\partial\rho^2}$$

$$=\left[-E_1\frac{1}{\rho^2}+2E_2+E_3(2\ln\rho+3)\right]+\left[2G_1\frac{1}{\rho^3}+G_2\frac{1}{\rho}+6G_3\rho\right]\cos\theta$$

$$+K_2\frac{1}{\rho}\theta\sin\theta+\sum_{m=2}^{n}[m(m+1)A_m\rho^{-m-2}+(2-m)(1-m)B_m\rho^{-m}$$

$$+m(m-1)C_m\rho^{m-2}+(m+2)(m+1)D_m\rho^m]\cos m\theta \qquad (3)$$

$$\sigma_\rho=\frac{1}{\rho}\frac{\partial\Phi}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\theta^2}$$

$$=\left[E_1\frac{1}{\rho^2}+2E_2+E_3(2\ln\rho+1)\right]+\left[-2G_1\frac{1}{\rho^3}+(G_2+2K_1)\frac{1}{\rho}+2K_2\frac{\ln\rho}{\rho}\right.$$

$$\left.+2G_3\rho\right]\cos\theta+K_2\frac{1}{\rho}\theta\sin\theta+\sum_{m=2}^{n}[-A_m m(1+m)\rho^{-m-2}+B_m(2-m-m^2)$$

$$\times\rho^{-m}+C_m m(1-m)\rho^{m-2}+D_m(2+m-m^2)\rho^m]\cos m\theta \qquad (4)$$

将（2）式代入（1）$_1$式，比较ρ的同次幂系数得，

$$\left.\begin{aligned}
&G_1\sin\alpha+3B_3\sin3\alpha=0\\
&-2B_2\sin2\alpha=0\\
&(G_2-K_2)\sin\alpha-K_2\alpha\cos\alpha=0\\
&2C_2\sin\alpha=q_0\\
&2G_3\sin\alpha+6C_3\sin3\alpha=q_1\\
&-m(1+m)A_m\sin m\alpha+(m+2)[-m-1]B_{m+2}\sin(m+2)\alpha=0\\
&m(m+1)D_m\sin m\alpha+(m+2)[m+1]C_{m+2}\sin(m+2)\alpha=q_m\\
&\qquad (m=2,\ 3\cdots,n)
\end{aligned}\right\} \qquad (5)$$

将（4）式中$\theta\sin\theta$展为级数

$$\theta\sin\theta=1-\frac{\cos\theta}{2}-\sum_{m=2}^{n}\frac{(-1)^m\cdot2}{m^2-1}\cos m\theta$$

并将（4）代入(1)$_3$式，得

$$\left.\begin{aligned}
&E_1+2E_2+E_3+K_2=0\\
&-2G_1+G_2+2K_1-\frac{K_2}{2}+2G_3=0\\
&-A_m m(1+m)+B_m(2-m-m^2)+C_m m(1-m)+D_m(2+m-m^2)\\
&-K_2\frac{(-1)^m\cdot2}{m^2-1}=0\ \ (m=2,3\cdots,n)
\end{aligned}\right\} \qquad (6)$$

由 $\frac{\partial \mu \rho}{\partial \rho} = \varepsilon_{\rho} = \frac{1}{E}(\sigma_{\rho} - \mu\sigma_{\theta})$

$$E\mu_{\rho} = \int(\sigma_{\rho} - \mu\sigma_{\theta})d\rho + f_1(\theta)$$

$$= -E_1(1+\mu)\frac{1}{\rho} + [2E_2(1-\mu) - 2E_3(1-\mu) + E_3(1-3\mu)]\rho + 2E_3(1-\mu)\rho\ln\rho$$

$$+\left\{G_1(1+\mu)\frac{1}{\rho^2} + [G_2(1-\mu) + 2K_1]\ln\rho + K_2\ln^2\rho + G_3(1-3\mu)\rho^2\right\}\cos\theta$$

$$+K_2(1-\mu)\ln\rho\theta\sin\theta + \sum_{m=2}^{n}\{A_m(1+\mu)m\rho^{-m-1} + B_m[m(1+\mu) - 2(\mu-1)]\rho^{1-m}$$

$$-C_m m(1+\mu)\rho^{m-1} + D_m[2(1-\mu) - m(1+\mu)]\rho^{m+1}\}\cos m\theta + f_1(\theta) \quad (7)$$

由本文第一节的假设，径向位移在内边界处为常数，取为 $\bar{u}_{\rho_0}$，解出 $f_1(\theta)$，将 $f_1(\theta)$ 代回(7)式

$$\mu_{\rho} = -\frac{E_1}{E}(1+\mu)\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right) + \frac{1}{E}[2E_2(1-\mu) - E_3(1+\mu)](\rho-\rho_0)$$

$$+\frac{2E_3}{E}(1-\mu)(\rho\ln\rho - \rho_0\ln\rho_0) + \frac{1}{E}\left\{G_1(1+\mu)\left(\frac{1}{\rho^2} - \frac{1}{\rho_0^2}\right) + [G_2(1-\mu)\right.$$

$$\left.+2K_1](\ln\rho - \ln\rho_0) + K_2(\ln^2\rho - \ln^2\rho_0) + G_3(1-3\mu)(\rho^2 - \rho_0{}^2)\right\}\cos\theta$$

$$+\frac{K_2(1-\mu)}{E}(\ln\rho - \ln\rho_0)\theta\sin\theta + \sum_{m=2}^{n}\frac{1}{E}\{A_m(1+\mu)m(\rho^{-m-1} - \rho_0{}^{-m-1})$$

$$+B_m[m(1+\mu) - 2(\mu-1)](\rho^{1-m} - \rho_0{}^{1-m}) - C_m m(1+\mu)(\rho^{m-1} - \rho_0{}^{m-1})$$

$$+D_m[2(1-\mu) - m(1+\mu)](\rho^{m+1} - \rho_0\rho^{m+1})\}\cos m\theta + \bar{u}_{\rho_0} \quad (8)$$

由 $\varepsilon_{\theta} = \frac{u_{\rho}}{\rho} + \frac{1}{\rho}\frac{\partial u_{\theta}}{\partial \theta}$

$$u_{\theta} = \int(\rho\varepsilon_{\theta} - u_{\rho})d\theta + f_2(\rho)$$

$$= \left[-\frac{E_1}{E}(1+\mu)\frac{1}{\rho} + \frac{2E_2}{E}(1-\mu)\rho + \frac{E_3}{E}2(1-\mu)\rho\ln\rho + \frac{E_3}{E}(3-\mu)\rho\right]\theta$$

$$+\frac{1}{E}\left[G_1 2(1+\mu)\frac{1}{\rho^2} + G_2(1-\mu) - K_1 2\mu - K_2 2\mu\ln\rho + G_2 2(3-\mu)\rho^2\right]\sin\theta$$

$$+\frac{K_2}{E}(1-\mu)(\sin\theta - \theta\cos\theta) + \sum_{m=2}^{n}\frac{1}{E}\{m(m+1)(1+\mu)A_m\rho^{-m-1}$$

$$+(m-1)[2(\mu-1) + m(1+\mu)]B_m\rho^{1-m} + m(m-1)(1+\mu)C_m\rho^{m-1}$$

$$+(m+1)[2(1-\mu) + m(1+\mu)]D_m\rho^{m+1}\}\frac{\sin m\theta}{m} - \frac{1}{E}\left\{-E_1(1+\mu)\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)\right.$$

$$\left.+[2E_2(1-\mu) - E_3(1+\mu)](\rho-\rho_0) + 2E_3(1-\mu)(\rho\ln\rho - \rho_0\ln\rho_0)\right\}\theta$$

$$-\frac{1}{E}\left\{G_1(1+\mu)\left(\frac{1}{\rho^2} - \frac{1}{\rho_0^2}\right) + [G_2(1-\mu) + 2K_1](\ln\rho - \ln\rho_0) + K_2(\ln^2\rho - \ln^2\rho_0)\right.$$

$$\left.+G_3(1-3\mu)(\rho^2 - \rho_0^2)\right\}\sin\theta - K_2(1-\mu)(\ln\rho - \ln\rho_0)(\sin\theta - \theta\cos\theta)$$

$$-\sum_{m=2}^{n}\frac{1}{E}\{A_m(1+\mu)m(\rho^{-m-1}-\rho_0^{-m-1})+B_m[m(1+\mu)-2(\mu-1)](\rho^{1-m}-\rho_0^{1-m})$$

$$-C_m m(1+\mu)(\rho^{m-1}-\rho_0^{m-1})+D_m[2(1-\mu)-m(1+\mu)](\rho^{m+1}-\rho_0^{m+1})\}$$

$$\frac{\sin m\theta}{m}-\bar{u}_{\rho 0}\theta+f_2(\rho) \quad (9)$$

由 $\theta=0$ 时 $u_\theta=0$

$f_2(\rho)=0$

将 $\ln\rho$ 展为幂级数 $(0<\rho\leqslant 1)$

$$\ln\rho=\sum_{m=1}^{n}-\frac{(1-\rho)^m}{m}$$

$$=\sum_{m=1}^{n}\Big[-\frac{1}{m}+\rho-\frac{(m-1)}{2!}\rho^2+\frac{(m-1)(m-2)}{3!}+\cdots$$

$$+(-1)^{i+1}\frac{(m-1)(m-2)\cdots(m-i+1)}{i!}\rho^i+\cdots+(-1)^{m+1}\frac{1}{m}\rho^m\Big]$$

$$=\tau_{m0}+\tau_{m1}\rho+\tau_{m2}\rho^2+\cdots\tau_{mi}\rho^i+\cdots+\tau_{mm}\rho^m \quad (10)$$

其中

$$\tau_{m0}=\sum_{m=1}^{n}-\frac{1}{m}$$

$$\tau_{m1}=\sum_{m=1}^{n}1$$

$$\tau_{m2}=\sum_{m=2}^{n}-\frac{(m-1)}{2!}$$

$$\tau_{mi}=\sum_{m=1}^{n}(-1)^{i+1}\frac{(m-1)(m-2)\cdots(m-i+1)}{i!}$$

$$\tau_{mm}=\sum_{m=1}^{n}(-1)^{m+1}\frac{1}{m}$$

将$\ln^2\rho$展为幂级数

设 $f(\rho)=\ln^2\rho$

$$f'(\rho)=\frac{2\ln\rho}{\rho}=\frac{2}{\rho}\sum_{m=1}^{n}\frac{-(1-\rho)^m}{m}$$

$$=\sum_{m=1}^{n}\Big[-\frac{2}{m\rho}+2-\frac{2(m-1)}{2!}\rho+\frac{2(m-1)(m-2)}{3!}\rho^2+\cdots+(-1)^{i+1}$$

$$\frac{2(m-1)(m-2)\cdots(m-i+1)}{i!}\rho^{i-1}+\cdots+(-1)^{m+1}$$

$$\frac{2(m-1)(m-2)\cdots 2\cdot 1}{m!}\rho^{m-1}\Big]$$

$$f(\rho)-f(\rho_0)=\int_{\rho_0}^{\rho}f'(\rho)d\rho$$

$$f(\rho)=\sum_{m=1}^{n}\Big[-\frac{2}{m}\ln\rho+2\rho-\frac{(m-1)}{2!}\rho^2+\frac{2(m-1)(m-2)}{3\times3!}\rho^3+\cdots$$

$$+(-1)^{i+1}\frac{2(m-1)\cdots(m-i+1)}{i\times i!}\rho^i+\cdots+(-1)^{m+1}\frac{2(m-1)(m-2)\cdots2\cdot1}{m\times m!}\rho^m\Big]$$

$$-H_{om} \tag{11}$$

其中 $H_{om}=\sum_{m=1}^{n}\Big[-\frac{2}{m}\ln\rho_0+2\rho_0-\frac{(m-1)}{2!}\rho_0^2+\frac{2(m-1)(m-2)}{3\times3!}\rho_0^3+\cdots$

$$+(-1)^{i+1}\frac{2(m-1)\cdots(m-i+1)}{i\times i!}\rho_0^i$$

$$+\cdots+(-1)^m\frac{2(m-1)(m-2)\cdots2\cdot1}{m\times m!}\rho_0^m\Big]+\ln^2\rho_0$$

将（10）代入（11）式 $\ln^2\rho=J_{km0}+J_{km1}\rho+J_{km2}\rho^2+\cdots+J_{kmi}\rho^i+\cdots+J_{kmm}\rho^m$ (12)

其中 $J_{km0}=I_{km0}-H_{om}$ $\quad J_{km1}=I_{km1}+I_{m1}$ $\quad J_{km2}=I_{km2}+I_{m2}$

$J_{kmi}=I_{kmi}+I_{mi}$ $\quad J_{kmm}=I_{kmm}+I_{mm}$

而 $I_{km0}=\sum_{k=1}^{n}\sum_{m=1}^{n}\frac{2}{Km}$ $\quad I_{km1}=\sum_{k=1}^{n}\left(-\frac{2}{K}\right)$ $\quad I_{km2}=\sum_{k=1}^{n}\sum_{m=1}^{n}\frac{m-1}{K}$

$$I_{kmi}=\frac{(-1)^i2(m-1)(m-2)\cdots(m-i+1)}{K\times i!}$$

$$I_{kmm}=\sum_{k=1}^{n}\sum_{m=1}^{n}\frac{(-1)^m2}{Km}\qquad I_{m1}=\sum_{m=1}^{n}2$$

$$I_{m2}=\sum_{m=1}^{n}-\frac{(m-1)}{2!}$$

$$I_{mi}=\sum_{m=1}^{n}(-1)^{i+1}\frac{2(m-1)(m-2)\cdots(m-i+1)}{i\times i!}$$

$$I_{mm}=\sum_{m=1}^{n}(-1)^{m+1}\frac{2}{m^2}$$

把（10），（12）式代入（9）式并利用$(1)_2$的位移边界条件式，由比较同类项系数得

$$\left.\begin{aligned}
&G_1\lambda_1+B_3\lambda_2=0\\
&2B_2(\mu-1)\sin2\alpha=0\\
&E_1\lambda_{11}-E_2\lambda_{11}+4E_3\alpha+C_2\lambda_{12}+K_2\lambda_{15}J_{m1}+K_1\lambda_{16}J_{m1}+G_2\lambda_{17}J_{m1}\\
&\quad+K_2\lambda_{18}J_{km1}=0\\
&G_3\lambda_{13}+C_3\lambda_{14}+K_2\lambda_{15}J_{m2}+K_1\lambda_{16}J_{m2}+G_2\lambda_{17}J_{m2}+K_2\lambda_{18}J_{km2}=0\\
&G_2\lambda_3+G_3\lambda_4+G_1\lambda_5-E_1\lambda_6+E_2\lambda_7+E_3\lambda_8+K_1\lambda_9+K_2\lambda_{10}-\bar{u}_{\rho_0}\alpha E+K_2\lambda_{15}J_{m0}\\
&\quad+K_1\lambda_{16}J_{m0}+G_2\lambda_{17}J_{m0}+K_2\lambda_{18}J_{km0}+\sum_{m=2}^{n}(A_mH_{1m}\rho_0^{-m-1}+B_mH_{2m}\rho_0^{1-m}\\
&\quad+C_mH_{3m}\rho_0^{m-1}+D_mH_{4m}\rho_0^{m+1})=0\\
&G_3\lambda_{13}+C_3\lambda_{14}=0\\
&A_mH_{5m}+B_{m+2}H_{6(m+2)}=0\\
&D_mH_{8m}+C_{m+2}H_{7(m+2)}=0
\end{aligned}\right\}\tag{13}$$

式中

$$\lambda_1=(1+\mu)\sin\alpha$$
$$\lambda_2=(3\mu-1)\sin3\alpha$$
$$\lambda_3=(1-\mu)(1+\ln\rho_0)\sin\alpha$$
$$\lambda_4=(1-3\mu)\rho_0^2\sin\alpha$$
$$\lambda_5=(1+\mu)\frac{1}{\rho_0^2}\sin\alpha$$
$$\lambda_6=(1+\mu)\frac{1}{\rho_0}\alpha$$
$$\lambda_7=2(1-\mu)\rho_0\alpha$$
$$\lambda_8=\rho_0\alpha[(1-\mu)\ln\rho_0-(1+\mu)]$$
$$\lambda_9=2\sin\alpha(\ln\rho_0-\mu)$$
$$\lambda_{10}=(1-\mu)(\sin\alpha-\alpha\cos\alpha)(1+\ln\rho_0)+\ln^2\rho_0\sin\alpha$$
$$\lambda_{11}=2(1-\mu)\alpha$$
$$\lambda_{12}=2(1+\mu)\sin2\alpha$$
$$\lambda_{13}=(5+\mu)\sin\alpha$$
$$\lambda_{14}=3(1+\mu)\sin3\alpha$$
$$\lambda_{15}=(1-\mu)\alpha\cos\alpha-(1+\mu)\sin\alpha$$
$$\lambda_{16}=-2\sin\alpha$$
$$\lambda_{17}=-(1-\mu)\sin\alpha$$
$$\lambda_{18}=-\sin\alpha$$
$$H_{1m}=(1+\mu)\sin m\alpha$$
$$H_{2m}=\frac{m(1+\mu)-2(\mu-1)}{m}\sin m\alpha$$
$$H_{3m}=-(1+\mu)\sin m\alpha$$
$$H_{4m}=\frac{2(1-\mu)-m(1+\mu)}{m}$$
$$H_{5m}=m(1+\mu)\sin m\alpha$$
$$H_{7m}=H_{5m}$$
$$H_{6m}=[m(1+\mu)-4]\sin m\alpha$$
$$H_{8m}=[m(1+\mu)+4]\sin m\alpha$$

为确定内边界径向位移$\bar{u}_{\rho0}$，需引入补充条件。根据本文假设，在内边界ρ_0处，σ_ρ均匀分布，其平均集度

$$\bar{p}=\frac{\int_0^{\alpha}\sigma_\rho|_{\rho=\rho_0}\rho_0 d\theta}{\int_0^{\alpha}\rho_0 d\theta}$$

$$=E_1\frac{1}{\rho_0^2}+2E_2+E_3(2\ln\rho_0+1)+\sum^{m}\Big[-2G_1\frac{1}{\rho_0^3}+G_2\frac{1}{\rho_0}+K_1\frac{2}{\rho_0}+K_2\frac{2\ln\rho_0}{\rho_0}$$

$$+2G_3\rho_0\Big]\frac{\sin\alpha}{\alpha}+K_2\frac{1}{\rho_0}\Big(\frac{\sin\alpha}{\alpha}-\cos\alpha\Big)+\sum_{m=2}^{n}[C_m m(1-m)\rho_0^{m-2}$$

$$+D_m(2+m-m^2)\rho_0^m-A_m m(1+m)\rho_0^{-m-2}+B_m(2-m-m^2)\rho_0^{-m}]\frac{\sin m\alpha}{m\alpha} \quad (14)$$

此时，$\bar{u}_{\rho_0}$的计算可按外周边自由，内周边受集度为p的均布荷载作用的圆环平面问题处理。由文〔5〕

$$\bar{u}_{\rho_0}=\bar{p}\beta$$

式中 $\beta=\dfrac{(\mu-1)\rho_0^3-(1+\mu)\rho_0}{E(1-\rho_0^2)}$

将（14）代入上式得补充方程

$$\frac{\bar{u}_{\rho_0}}{\beta}-E_1\frac{1}{\rho_0^2}-2E_2-E_3(2\ln\rho_0+1)=\Big[-2G_1\frac{1}{\rho_0^3}+G_2\frac{1}{\rho_0}+K_1\frac{2}{\rho_0}+K_2\frac{2\ln\rho_0}{\rho_0}$$

$$+2G_3\rho_0\Big]\frac{\sin\alpha}{\alpha}+K_2\frac{1}{\rho_0}\Big(\frac{\sin\alpha}{\alpha}-\cos\alpha\Big)+\sum_{m=2}^{n}[C_m m(1-m)\rho_0^{m-2}+D_m(2$$

$$+m-m^2)\rho_0^m-A_m m(1+m)\rho_0^{-m-2}+B_m(2-m-m^2)\rho_0^{-m}]\frac{\sin m\alpha}{m\alpha} \quad (15)$$

解由（5），（6），（13），（15）组成的代数方程组得

$$\left.\begin{aligned}
&G_1=B_2=B_3=0\\
&A_m=B_m=0\quad(m=2,\ 3\cdots,n)\\
&C_2=q_0\frac{1}{2\sin\alpha}\\
&G_3=q_1g_1\\
&C_3=q_1g_2\\
&K_2=q_1g_{3m}\\
&G_2=q_1g_{4m}\\
&K_1=q_1g_{5m}\\
&E_1=q_0t_0+q_1t_{1m}+\sum_{m=2}^{n}q_mt_{3m}\\
&E_3=q_0t_4+q_1t_5+\sum_{m=2}^{n}q_mt_{6m}\\
&E_2=q_0t_7+q_1t_{8m}+\sum_{m=2}^{n}q_mt_{9m}\\
&\bar{u}_{\rho_0}=q_0u_0+q_1u_1+\sum_{m=2}^{n}q_mu_m\\
&D_m=q_mL_{m1}\\
&C_{m+2}=q_mL_{m2}\quad(m=2,\ 3\cdots,n)
\end{aligned}\right\}\quad(16)$$

式中

$$g_1=\frac{(1+\mu)}{-(9+\mu)\sin\alpha}$$

$$g_2=\frac{(5+\mu)}{3(9+\mu)\sin 3\alpha}$$

$$g_{3m}=\frac{2g_2\lambda_{14}-g_1(\lambda_{13}+2\lambda_{16}J_{m2})}{\lambda_{19}\lambda_{16}J_{m2}-2\lambda_{20m}}$$

$$g_{4m}=\lambda_{19}g_{3m}$$

$$g_{5m}=-g_1-\frac{g_{3m}\lambda_{19}}{2}$$

$$t_0=\frac{2s_0\lambda_{26}-2s_5\lambda_{24}}{3\lambda_{11}\lambda_{26}-\lambda_{25}\lambda_{24}}$$

$$t_{1m}=\frac{s_{11m}\lambda_{26}-s_{12m}\lambda_{24}}{3\lambda_{11}\lambda_{26}-\lambda_{25}\lambda_{24}}$$

$$t_{3m}=\frac{-2s_{10m}\lambda_{24}}{3\lambda_{11}\lambda_{26}-\lambda_{25}\lambda_{24}}$$

$$t_4=\frac{2s_0-3\lambda_{11}t_0}{\lambda_{24}}$$

$$t_{5m}=\frac{s_{11m}-3\lambda_{11}t_{1m}}{\lambda_{24}}$$

$$t_{6m}=\frac{-3\lambda_{11}t_{3m}}{\lambda_{24}}$$

$$t_7=-\frac{t_0+t_4}{2}$$

$$t_{8m}=-\frac{g_{3m}+t_{1m}+t_{5m}}{2}$$

$$t_{9m}=-\frac{t_{3m}+t_{6m}}{2}$$

$$u_0=(-s_2-\lambda_6 t_0+\lambda_7 t_7+\lambda_8 t_4)\frac{1}{Ea}$$

$$u_1=(-\lambda_6 t_1+\lambda_7 t_{8m}+\lambda_8 t_{5m}-s_{3m})\frac{1}{Ea}$$

$$u_m=(-\lambda_6 t_{3m}+\lambda_7 t_{9m}+\lambda_8 t_{6m}-s_{4m})\frac{1}{Ea}$$

$$L_{m1}=\frac{(1+\mu)}{-4(m+1)\sin m\alpha}$$

$$L_{m2}=\frac{[m(1+\mu)+4]}{4(m+1)(m+2)\sin(m+2)\alpha}$$

其中

$$\lambda_{19}=\frac{1}{2}+\alpha\mathrm{ctg}\alpha$$

$$\lambda_{20m}=\lambda_{15}J_{m2}+\lambda_{18}J_{km2}+(1+\alpha\mathrm{ctg}\alpha)\lambda_{17}J_{m2}$$

$$\lambda_{21}=\frac{-E\alpha\beta}{\rho_0^2}-\lambda_6$$

$$\lambda_{22}=-2E\alpha\beta+\lambda_7$$

$$\lambda_{23}=-E\alpha\beta(2\ln\rho_0+1)+\lambda_8$$

$$\lambda_{24}=8\alpha+\lambda_{11}\qquad \lambda_{25}=2\lambda_{21}-\lambda_{22}$$

$$\lambda_{26}=2\lambda_{23}-\lambda_{22}\qquad s_0=-\frac{\lambda_{12}}{2\sin\alpha}$$

$$s_2=(1+\mu)\cos\alpha\cdot\rho_0$$

$$s_{3m}=g_1(1+\mu)\sin3\alpha-\lambda_3g_{4m}-\lambda_4g_1-g_{5m}(\lambda_9+\lambda_{18}J_{m0})$$
$$-g_{3m}(\lambda_{10}+\lambda_{18}J_{km0}+\lambda_{15}J_{m0})-\lambda_{17}g_{4m}J_{m0}$$

$$s_{4m}=-(L_{m1}H_{4m}+L_{m2}H_{3(m+2)})\rho_0^{m+1}$$

$$s_5=-\frac{\beta\cos\alpha}{\alpha}$$

$$s_{8m}=\Big[g_1\frac{2\sin\alpha}{\alpha}\rho_0-2g_2\rho_0\frac{\sin3\alpha}{\alpha}+g_{3m}\Big(\frac{2\ln\rho_0\sin\alpha}{\rho_0\alpha}$$
$$+\frac{\sin\alpha-\alpha\cos\alpha}{\rho_0\alpha}\Big)+g_{4m}\frac{\sin\alpha}{\rho_0\alpha}+g_{5m}\frac{2\sin\alpha}{\rho_0\alpha}\Big]\beta$$

$$s_{7m}=L_{m1}H_{9m}+L_{m2}H_{10m}$$

$$H_{9m}=(2+m-m^2)\frac{\sin m\alpha}{m\alpha}\rho_0^m\beta$$

$$H_{10m}=-(m+1)(m+2)\frac{\sin(m+2)\alpha}{m\alpha}\rho_0^m\beta$$

$$s_8=E\alpha s_5+s_2\qquad s_{9m}=E\alpha s_6+s_{3m}$$

$$s_{10m}=E\alpha s_{7m}+s_{4m}\qquad s_{11m}=2s_{1m}-g_{3m}\lambda_{11}$$

$$s_{12m}=2s_{9m}+g_1\lambda_{22}$$

3 径向位移和协调条件

将（16）式代入（8）式得出径向位移的一般式

$$u_\rho=q_0F_0+q_1F_1+\sum_{m=2}^{n}q_mF_m \tag{27}$$

式中

$$F_1=\frac{1}{E}\Big\{-t_0(1+\mu)\Big(\frac{1}{\rho}-\frac{1}{\rho_0}\Big)+\Big[2t_7(1-\mu)-t_4(1+\mu)-\frac{(1+\mu)}{\sin\alpha}\cos2\theta\Big]$$
$$\times(\rho-\rho_0)+2t_4(1-\mu)(\rho\ln\rho-\rho_0\ln\rho_0)\Big\}+u_0$$

$$F_2=\frac{1}{E}\Big\{-t_1(1+\mu)\Big(\frac{1}{\rho}-\frac{1}{\rho_0}\Big)+[2t_{8m}(1-\mu)-t_{5m}(1+\mu)](\rho-\rho_0)$$
$$+2t_{5m}(1-\mu)(\rho\ln\rho-\rho_0\ln\rho_0)+[(g_{4m}(1-\mu)+2g_{5m})\cos\theta+g_{3m}$$
$$\times(1-\mu)\theta\sin\theta](\ln\rho-\ln\rho_0)+g_{3m}(\ln^2\rho-\ln^2\rho_0)\cos\theta+[g_1(1-3\mu)$$

$$\times\cos\theta-3g_2(1+\mu)\cos3\theta](\rho^2-\rho_0^2)\Big\}+u_1$$

$$F_m=\frac{1}{E}\Big\{-t_{3m}(1+\mu)\Big(\frac{1}{\rho}-\frac{1}{\rho_0}\Big)+[2t_{9m}(1-\mu)-t_{8m}(1+\mu)](\rho-\rho_0)$$

$$+2t_{6m}(1-\mu)(\rho\ln\rho-\rho_0\ln\rho_0)+[L_{m1}(2(1-\mu)-m(1+\mu))\cos m\theta$$

$$-L_{m2}(m+2)(1+\mu)\cos(m+2)\theta](\rho^{m+1}-\rho_0^{m+1})\Big\}+u_m$$

将底板对桁架的反作用力化为其下弦的节点力F_i（如图3）

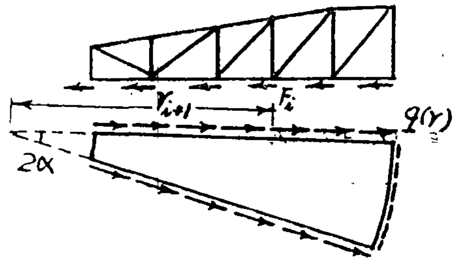

图3 底板、桁架间的相互作用力

$$F_i=\int_{\frac{r_i-r_{i-1}}{2}}^{\frac{r_{i+1}-r_i}{2}}\sum_{m=0}^{n}q_m r^m dr \qquad (\text{式中}r_i=\rho_i R)$$

$$=\sum_{m=0}^{n}q_m\frac{r_{i+1}'^{\,m+1}-r_i'^{\,m+1}}{m+1}$$

式中

$$r_i'=\frac{r_i-r_{i-1}}{2},\quad r_{i+1}'=\frac{r_{i+1}-r_i}{2}\quad (i=1,\ 2\cdots,\ n)$$

当$i=n$时，$r_{i+1}'=r_n$

由（14）式底板对中心筒壳的作用力p

$$p=q_0p_0+q_1p_1+\sum_{m=2}^{n}q_mp_m$$

式中

$$p_0=\frac{t_0}{\rho_0^2}+2t_7+t_4(2\ln\rho_0+1)-\frac{\cos\alpha}{\alpha}$$

$$p_1=\frac{t_1}{\rho_0^2}+2t_{8m}+t_5(2\ln\rho_0+1)+\Big(g_{4m}\frac{1}{\rho_0}+g_{5m}\frac{2}{\rho_0}+g_{6m}\frac{2\ln\rho_0}{\rho_0}$$

$$+2g_1\rho_0\Big)\frac{\sin\alpha}{\alpha}+g_{3m}\frac{1}{\rho_0}\Big(\frac{\sin\alpha}{\alpha}-\cos\alpha\Big)-2g_2\rho_0\frac{\sin3\alpha}{\alpha}$$

$$p_m=t_{3m}\frac{1}{\rho_0^2}+2t_{9m}+t_{6m}(2\ln\rho_0+1)+L_{m1}\rho_0^m\frac{\sin m\alpha}{m\alpha}$$

$$-L_{m2}(m+1)(m+2)\rho_0^m\frac{\sin(m+2)\alpha}{(m+2)\alpha}$$

根据p可以计算出中心筒壳在p和其它荷载作用下与底板联接处的径向位移u_c。由桁架下弦节点力F_i，可以计算出桁架在F_i和其它荷载作用下下弦各节点的径向位移u_{ti}。根据底板与中心筒壳和桁架的径向位移协调条件，得出方程组，

$$u_{\rho_0}=u_c$$

$$u_{\rho_i}=u_{ti}\qquad (i=1,\ 2\cdots,n)$$

解此方程组确定q_0，$q_1\cdots$，q_n，可以确定问题的完整解答。

4 结 语

本文从理论上讨论了部分环形圆板受图2示约束及边缘荷载时的准确解。它既可以单独使用，处理有关问题，又可以在大型容器活塞构架的整体分析中，用来考虑底板与桁架间的剪切效应。我们的出发点主要是后者。由于大型容器活塞的整体分析，还涉及到中心圆筒、桁架以及底板弯曲等计算问题，这些已超出了本文范围，因此，本文的具体应用，将在讨论活塞整体分析时，另文介绍。

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THE THEORETICAL ANALYSIS OF RADIAL DISPLACEMENT AND COMPATIBLE CONDITIONS OF BOTTOM PLATE OF PISTON FRAME IN LARGE CONTAINER

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ABSTRACT Taking account of shear action between the piston truss and the bottom plate, this paper assumes the bottom plate to be a part-ring plate and supposes that the shear distribution follows the power series law, thus making the problem to be a problem of mixed boundary condition of part-ring plate. In the paper, the solution of this elastic plane problem is derived. The general expressions of radial displacement of bottom plate are given. The compatible conditions both at the nodes of the lower chord of the truss and at the central cylindrical shell are analyzed.

KEW WORDS sector of ring plate, plane problem, power series