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# 直墙拱形巷道围岩应力场分析



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摘要:为得出直墙拱形巷道围岩应力分布规律,应用复变函数弹性理论推导了直墙拱形巷道围岩应力分布的解析表达式。对直墙拱形巷道边界的围岩应力和巷道水平线方向的围岩应力分布规律进行分析,并考虑直墙拱形巷道断面高宽比和侧压系数对其影响规律。研究表明:在不同巷道断面高宽比、侧压系数下,直墙拱形巷道围岩应力集中区域主要集中在直墙底部底角处、拱形顶板中点附近和底板中部3个位置。不同巷道断面高宽比下,直墙拱形巷道沿水平线的应力分布规律基本相同。侧压系数大于1时,采用巷道断面高宽比小于1较有利于巷道稳定;侧压系数小于等于1时,采用巷道断面高宽比大于1较有利于巷道稳定。

关键词:直墙拱形巷道;复变函数;围岩应力

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# Analysis on surrounding rock stress field of vertical wall archy roadway

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Abstract: The conformal transformation and elastic theory of complex function are used to derive the analytical expression of surrounding rock stress distribution of vertical wall archy roadway. The distribution laws of roadway boundary surrounding rock stress and the stress along horizontal line are analyzed. Also the impact of different aspect ratios of roadway cross-section and different lateral pressure coefficients on stress field of surrounding rocks are studied. The results showe that: with different aspect ratios of roadway cross-section and different lateral pressure coefficients, three locations of vertical wall archy roadway surrounding rocks are main stress concentration area concentrated. The three locations are the basic angle at bottom of vertical wall, the location near midpoint of archy roof and the center of floor. Under different aspect ratios of roadway cross-section, the distribution laws of stress along horizontal line of vertical wall archy roadway are the same basically. When the lateral pressure coefficient is greater than one, roadway is stable with aspect ratio of roadway cross-section less than one. While, when the lateral pressure coefficient is not greater than 1, roadway is stable with aspect ratio of roadway cross-section for a stress concentrated of stress coefficient is not greater than one.

Key words: vertical wall archy roadway; complex function; surrounding rock stress

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煤矿巷道围岩应力大小和规律是巷道支护方 式选取的重要依据之一。围岩应力大小不仅与煤 矿采深、侧压系数等有关,还与巷道断面形状等相 关(即使相同围岩条件下,围岩应力分布规律和围 岩变形破坏规律也因巷道断面形状不同而不同)。 直墙拱形断面巷道的断面由下部分矩形和上部分 拱形组成,长期实践证明,直墙拱形断面巷道具有 较好的稳定性,所以服务年限较长的巷道一般均采 用直墙拱形断面巷道。了解和掌握巷道围岩应力 分布规律对合理选择巷道支护方式具有重要的理 论意义和实际应用价值。对于常规的圆形、椭圆形 等巷道的围岩应力可以采用 Cauchy 积分法或幂级 数法方便解出[1-3],但复杂巷道围岩应力公式需借 助复变函数弹性理论[4-6]及映射函数[7-9]。朱大勇 等<sup>[10]</sup>求解了矩形断面围岩应力弹性解,赵凯等<sup>[11]</sup> 利用多角形法得出了矩形硐室围岩应力,王润 富<sup>[12]</sup>、刘金高等<sup>[13]</sup>求解了梯形孔口的应力,汤澄 波等[14]、祝江鸿[15]分别利用复变函数法求解了天 幕线拱形围岩应力和两个表示断面围岩应力的解 析函数通式,但前提是能给出复杂断面映射函数。 针对直墙拱形巷道围岩应力研究较少,对其他复 杂巷道研究大多也只给出复杂巷道应力解析的 隐函数,未给出巷道应力分布情况,也未对巷道 断面高宽比和侧压系数对其影响规律进行深入 分析。

# 1 模型建立

为简化计算,将巷道整个断面简化为以 y 轴为 对称轴的六边形,无支护阻力,远场铅垂应力为  $\sigma_v$ , 水平应力为  $\sigma_h = k\sigma_v$ , k 为侧压系数,不计体力,计算 模型如图 1 所示。设顶板宽为 $\overline{B_1A_1} = 2a$ ,底板宽为  $\overline{B_3A_3} = 2b$ ,断面总高度为 $\overline{DN} = 2h$ ,拱的高度为  $\overline{OD} = h_b$ ,直墙高为 $\overline{ON} = h_b = 2h - h_a$ ,直墙拱形断面 高宽比  $c_0 = h/b$ 。以拱顶与直墙交界线和直墙拱形 巷道断面的对称轴为 x, y 轴建立平面直角坐标系。 以 y 轴为对称轴的六边形的顶点从右上方顺时针依 次为  $A_1$ , $A_2$ , $A_3$ , $B_3$ , $B_2$ , $B_1$ 。 $A_1$ , $A_2$ , $A_3$ 处的外角分别 为  $\vartheta_1 = \phi_1$ , $\vartheta_2 = \phi_2$ , $\vartheta_3 = \phi_3 = \frac{3\pi}{2}$ 。由几何关系可得出  $\vartheta_1 = \frac{3\pi}{2} - \arctan \frac{b-a}{2h-H}$ ,  $\vartheta_2 = \pi + \arctan \frac{b-a}{2h-H}$  (1)



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图 1

模型图

Fig. 1 diagram of model

# 2 直墙拱形巷道围岩应力解析

#### 2.1 保角变换

首先将直墙拱形巷道围岩映射为单位圆形巷道 围岩,即把图 1 中的六边形的外域部分映射为图 2 的单位圆外域部分。映射后  $A_1$ 、 $A_2$ 、 $A_3$ 、 $B_3$ 、 $B_2$ 、 $B_1$ 分别  $a_{1t}$ 、 $a_{2t}$ 、 $a_{3t}$ 、 $b_{3t}$ 、 $b_{2t}$ 、 $b_{1t}$ 与对应,如图 2。则有  $a_{1t}$ =ie<sup>-ik<sub>1</sub>π</sup>,  $a_{2t}$ =ie<sup>-ik<sub>2</sub>π</sup>,  $a_{3t}$ =ie<sup>-ik<sub>3</sub>π</sup>,  $b_{1t}$ =ie<sup>-ik<sub>1</sub>π</sup>,  $b_{2t}$ =ie<sup>-ik<sub>2</sub>π</sup>, $b_{3t}$ =ie<sup>-ik<sub>3</sub>π</sup>。其中  $K_1$ 、 $K_2$ 、 $K_3$ 由六边形 的尺寸确定。

由 $\partial_1 = \phi_1, \partial_2 = \phi_2, \partial_3 = \phi_3 = \frac{3\pi}{2}$ 根据许瓦兹-克里 斯托夫的多角映射理论,可得直墙拱形巷道映射函 数为

$$z = \omega(t) = R \int_{t} \left( \frac{1 - i\lambda_1}{\tau} - \frac{1}{\tau^2} \right)^{\frac{\vartheta_1}{\pi} - 1} \cdot \left( \frac{1 - i\lambda_2}{\tau} - \frac{1}{\tau^2} \right)^{\frac{\vartheta_2}{\pi} - 1} \left( \frac{1 - i\lambda_3}{\tau} - \frac{1}{\tau^2} \right)^{\frac{\vartheta_3}{\pi} - 1} d\tau \quad (2)$$

式中: $\lambda_1 = 2 \cos \kappa_1 \pi, \lambda_2 = 2 \cos \kappa_2 \pi, \lambda_3 = 2 \cos \kappa_3 \pi$ 。

作变量替换  $\tau = \frac{1}{T}$ ,可将 t 平面的单位圆外域映 射为  $\xi$  平面的单位圆内域,得映射函数

$$z = \omega(\zeta) = -R \int_{\zeta} \frac{1}{T^2} (1 - \mathrm{i}\lambda_1 T - T^2)^{\frac{\vartheta_1}{\pi} - 1} \cdot$$

$$(1 - i\lambda_{2}T - T^{2})^{\frac{\vartheta_{2}}{\pi} - 1}(1 - i\lambda_{3}T - T^{2})^{\frac{\vartheta_{3}}{\pi} - 1}dT \quad (3)$$

$$\bigcup_{T} f(T) = f_{1}(T)f_{2}(T)f_{3}(T), f_{1}(T) = (1 - i\lambda_{1}T - T^{2})^{\varphi_{1}}, \varphi_{1} = \frac{\vartheta_{1}}{\pi}, f_{2}(T) = (1 - i\lambda_{2}T - T^{2})^{\varphi_{2}},$$

$$\Phi_{2} = \frac{\vartheta_{2}}{\pi} - 1, f_{3}(T) = (1 - i\lambda_{3}T - T^{2})^{\varphi_{3}}, \Phi_{3} = \frac{\vartheta_{3}}{\pi} - 1.$$

将以下 3 个函数在零点的邻域内展开为级数,得

$$R\left(\frac{1}{\zeta} - B_1 \ln \zeta - B_2 \zeta - \frac{D_3}{2} \zeta^2 - \frac{D_4}{3} \zeta^3 - \frac{D_5}{4} \zeta^4 - \cdots\right)$$
(4)

为保证上式为单值函数,应有  $B_1 = C_{11} + C_{21} + C_{31} = 0$ 。

由 
$$C_{11} = -i\lambda_1 \Phi_1$$
,  $C_{21} = -i\lambda_2 \Phi_2$ ,  $C_{31} = -i\lambda_3 \Phi_3$ ,  
 $\lambda_1 = 2 \cos \kappa_1 \pi$ ,  $\lambda_2 = 2 \cos \kappa_2 \pi$ ,  $\lambda_3 = 2 \cos \kappa_3 \pi$ , 得

 $Φ_1 \cos \kappa_1 \pi + Φ_2 \cos \kappa_2 \pi + Φ_3 \cos \kappa_3 \pi = 0$  (5) 采用试算法,先根据巷道几何尺寸确定  $\kappa_1 \pi$ 、  $\kappa_2 \pi$ ,然后可由式(5)确定出  $\kappa_3 \pi$ 。

整理后得出直墙拱形巷道外域到单位圆内域的 映射函数为

$$z = \omega(\zeta) = -R\left(\zeta - \frac{B_2}{\zeta} - \frac{B_3}{2\zeta^2} - \frac{B_4}{3\zeta^3} - \frac{B_5}{4\zeta^4} - \cdots\right)$$
(6)

根据范广勤研究结果,取前 5 项即可达到足够的精度,且  $B_2$ 、 $B_4$ 为实数, $B_3$ 、 $B_5$ 为纯虚数,因此,令

$$B_{2} = b_{2} , B_{4} = b_{4} , B_{3} = ib_{3} , B_{5} = ib_{5} , \mathbb{R} \text{ if } 5 \ \mathfrak{I}, \mathbb{R}$$

$$z = \omega(\zeta) = -R\left(\frac{1}{\zeta} - b2\zeta - \frac{ib_{3}}{2}\zeta^{2} - \frac{ib_{4}}{3}\zeta^{3} - \frac{ib_{5}}{4}\zeta^{4}\right)$$
(7)

其中,  $b_i$  由  $c_{ij}$  表示,  $c_{l1} = -\lambda_l \Phi_l$ ,  $c_{l2} = -\Phi_l \Big[ 1 + \frac{\lambda_l^2}{2} (\Phi_l - 1) \Big]$ ,  $c_{l3} = \lambda_l \Phi_l (\Phi_l - 1) \Big[ 1 + \frac{\lambda_l^2}{6} (\Phi_l - 2) \Big]$ ,  $c_{l4}$ =  $\lambda_l \Phi_l (\Phi_l - 1) \Big[ \frac{1}{2} + \frac{\lambda_l^2}{2} (\Phi_l - 2) + \frac{\lambda_l^4}{24} (\Phi_l - 2) (\Phi_l - 2) \Big]$ ,  $c_{l5} = \lambda_l \Phi_l (\Phi_l - 1) (\Phi_l - 2) \Big[ \frac{1}{2} + \frac{\lambda_l^2}{6} (\Phi_l - 3) + \frac{\lambda_l^4}{120} (\Phi_l - 3) (\Phi_l - 4) \Big]$ , l = 1, 2, 3.

在 $\zeta$ 平面的单位圆周上, $\rho = 1$ ; $\zeta = e^{i\varphi} = \cos\varphi + i \sin\varphi$ 。在 z 平面的六边形上。 $z_0 = x_0 + i y_0$ 。代入映 射函数得

$$x_{0} = R\left(\cos\varphi - b_{2}\cos\varphi + \frac{b_{3}}{2}\sin 2\varphi - \frac{b_{4}}{3}\sin 3\varphi + \frac{b_{5}}{4}\sin 4\varphi\right)$$
$$y_{0} = -R\left(\sin\varphi - b_{2}\sin\varphi + \frac{b_{3}}{2}\cos 2\varphi - \frac{b_{4}}{3}\sin 3\varphi + \frac{b_{5}}{4}\cos 4\varphi\right)$$
(8)

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ζ平面上点(ρ = 1, φ = 0), 与 z 平面巷道直墙 上一点相对应, 其坐标为  $x_0 = b, y_0 = y_p$ , 得  $b = R\left(1 - b_2 - \frac{b_4}{3}\right), y_p = - R\left(\frac{b_3}{2} + \frac{b_5}{4}\right), R = \frac{b}{1 - b_2 - \frac{b_4}{3}}$ 。当断面高宽比分别取 0.5、1、1.5 时, 保角变换参数如图 3。





#### 2.2 由边界条件确定

远场铅垂应力为 ov、水平应力为 oh = kov, 所以

$$\alpha = \frac{(1+k)\sigma_{v}}{4}, \alpha_{1} = \frac{(1+k)\sigma_{v}}{2}, \beta_{1} = 0$$

因无支护阻力,
$$X_s = Y_s = 0, P_x = P_y = 0, 则有$$
  
 $f_0 = -2\alpha\omega(\sigma) - (\alpha_1 - i\beta_1)\overline{\omega(\sigma)}$  (9)

2.3 确定 φ<sub>0</sub>(ζ)、ψ<sub>0</sub>(ζ)

$$\varphi_{0}(\zeta) = a_{1}\zeta + ia_{2i}\zeta^{2} + 2aR\left(\frac{b_{4}}{3}\zeta^{3} + \frac{ib_{5}}{4}\zeta^{4}\right)(10)$$

$$\psi_{0}(\zeta) = -\frac{ib_{5}a_{1}}{4\zeta^{2}} + \left(\frac{b_{5}a_{2i}}{2} + \frac{b_{4}a_{1}}{3}\right)\frac{1}{\zeta} +$$

$$R(a_{1}b_{2} - 2a)\zeta + a_{1}R\left(\frac{ib_{3}}{2}\zeta^{2} + \frac{b_{4}}{3}\zeta^{3} + \frac{ib_{5}}{4}\zeta^{4}\right) +$$

$$\zeta^{3} - b_{2}\zeta + \frac{ib_{3}}{2} - \frac{b_{4}}{3\zeta} + \frac{ib_{5}}{4\zeta^{2}} + \zeta(\zeta) = \zeta(11)$$

$$\frac{2}{1+b_2\zeta^2+ib_3\zeta^3+b_4\zeta^4+ib_5\zeta^5}\varphi'_0(\zeta) \quad (11)$$

$$\varphi(\zeta) = \alpha \omega(\zeta) + \varphi_0(\zeta) =$$

$$\alpha R\left(\frac{1}{\zeta} - b_2 \zeta - \frac{\mathrm{i}b_3}{2} \zeta^2 - \frac{b_4}{3} \zeta^3 - \frac{\mathrm{i}b_5}{4} \zeta^4\right) + \varphi_0(\zeta)$$
(12)

$$\psi(\zeta) = (\alpha_1 + \mathrm{i}eta_1)\omega(\zeta) + \psi_0(\zeta) =$$

$$\alpha_{1}R\left(\frac{1}{\zeta}-b_{2}\zeta-\frac{ib_{3}}{2}\zeta^{2}-\frac{b_{4}}{3}\zeta^{3}-\frac{ib_{5}}{4}\zeta^{4}\right)+\psi_{0}(\zeta)$$
(13)

## 2.5 确定应力分量

 $\Phi(\zeta) = \varphi'(\zeta) / \omega'(\zeta), \Psi(\zeta) = \psi'(\zeta) / \omega'(\zeta), 可得$ 直墙拱形巷道围岩应力分量的曲线坐标表达式

$$\sigma_{\varphi} + \sigma_{\rho} = 4Re\left[\Phi(\zeta)\right] = 4Re\left[\alpha + \frac{\varphi_{0}'(\zeta)}{\omega'(\zeta)}\right] (14)$$

$$\sigma_{\varphi} - \sigma_{\rho} + 2i\tau_{\rho\varphi} = \frac{2\zeta^{2}}{\rho^{2}\omega'(\zeta)} \left[\overline{\omega(\zeta)}\Phi'(\zeta) + \omega'(\zeta)\Psi(\zeta)\right] =$$

$$\frac{2\zeta^{2}}{\rho^{2}\overline{\omega'(\zeta)}} \begin{bmatrix} \overline{\omega(\zeta)}\left[\frac{\varphi_{0}''(\zeta)}{\omega'(\zeta)} - \frac{\varphi_{0}'\omega''(\zeta)}{\omega'(\zeta)}\right]^{2} \end{bmatrix} + \right] =$$

$$\frac{2\zeta^{2}}{\rho^{2}\overline{\omega'(\zeta)}} \begin{bmatrix} \overline{\omega(\zeta)}\left[\frac{\varphi_{0}''(\zeta)}{\omega'(\zeta)} - \frac{\varphi_{0}'\omega''(\zeta)}{\omega'(\zeta)}\right] \end{bmatrix} =$$

$$\frac{2\zeta^{2}}{\rho^{2}\overline{\omega'(\zeta)}} \begin{bmatrix} \overline{\omega(\zeta)}\left[\frac{\varphi_{0}''(\zeta)}{\omega'(\zeta)} - \frac{\varphi_{0}'\omega''(\zeta)}{\omega'(\zeta)}\right] + \right]$$

$$(15)$$

# 3 直墙拱形巷道围岩应力分析

# 3.1 直墙拱形巷道边界应力分布规律

巷道边界处, $\rho = 1$ , $\zeta = e^{i\varphi}$ , $\sigma_{\rho} = \tau_{\rho\rho} = 0$ 。取a = 1 m,b = 2 m,当断面高宽  $c_0$ 比分别取 0.5、1、1.5 时,得直墙拱形巷道周边应力分布规律分别如图 4 (a)、(b)、(c),-90°~90°对应底板中点到顶板中点 围岩边界。



从图 4(a)可得:巷道断面高宽比为 0.5 情况下,侧压系数大于 1 时围岩边界环向应力有相同变化趋

势,即底板中点到直墙底部再到直墙顶部的边界环向 应力先增后减,在直墙底部底角处区域出现较大应力 集中;直墙顶部到拱形顶板中点的边界环向应力先减 后增再减。侧压系数小于1时,围岩边界环向应力也 有相同变化趋势,即底板中点和顶板中点附近均出现 拉应力,直墙底部到直墙顶部的边界环向应力较为恒 定;直墙顶部到拱形顶板中点的边界环向应力先增后 减然后变为拉应力。应力集中区域主要集中在直墙 底部底角处和拱形顶板中点附近。

从图 4(b)可得:巷道断面高宽比为 1 情况下, 侧压系数大于 1 时,围岩边界环向应力有相同变化 趋势,即底板中点附近均出现拉应力,底板中点到直 墙底部再到直墙顶部的边界环向应力先由拉应力变 为压应力,然后增加后减小;直墙顶部到拱形顶板中 点的边界环向应力先减后增再减。侧压系数小于 1 时,围岩边界环向应力也有相同变化趋势,即顶板 中点附近均出现拉应力,底板中点到直墙底部再到 直墙顶部的边界环向应力先由逐渐减小;直墙顶部 到拱形顶板中点的边界环向应力先增后减然后变为 拉应力。应力集中区域主要集中在底板中部和拱形 顶板中点附近,且拱形顶板应力集中系数小于底板。

从图 4(c)可得:巷道断面高宽比为 1.5 情况下, 巷道围岩应力分布规律与巷道断面高宽比为 1 情况 基本相同,不同之处在于拱形顶板应力集中系数大 于底板,围岩应力分布比巷道断面高宽比为 1 时 较好。

3.2 直墙拱形巷道沿水平线的应力分布规律

取  $\varphi = 0$ , $\zeta = \rho$ 为直墙拱形巷道水平线位置,由  $x = R(1 + c_1 \rho^2 + c_3 \rho^4) / \rho$  可将曲线坐标表示的应力 分量表达式转换为直角坐标表示。取 a = 1 m,b = 2m,断面高宽比  $c_0$  分别取 0.5、1、1.5 时,可得沿 x轴围岩应力分布规律,如图 5(a)、(b)、(c)。



图 5 直角坐标下巷道周边应力分布规律

Fig. 5 Stress distribution rule of perimeter under rectangular coordinate

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从图 5 可得:1)不同巷道断面高宽比下,直墙拱 形巷道沿水平线的应力分布规律基本相同。2)侧压 系数大于1时,不同巷道断面高宽比的环向应力均 随至巷道边界距离增大而迅速增大,在距离巷道边 界 2~4 m 后达到稳定;侧压系数小于1时,不同巷 道断面高宽比的环向应力均随至巷道边界距离增大 而先增大后减小,在距离巷道边界1m左右达到最 大值,然后较小并在距离巷道边界 2~4 m 后达到稳 定。3)侧压系数大于1时,不同巷道断面高宽比的 径向应力均随至巷道边界距离增大而先减小后变为 拉应力然后增加,在距离巷道边界 4~6 m 后达到稳 定;侧压系数小于1时,不同巷道断面高宽比的径向 应力均随至巷道边界距离增大而增大,在距离巷道 边界 2~4 m 后达到稳定。4)直墙拱形巷道边界 3 m范围内出现了剪应力,剪应力随至巷道边界距离 增大而迅速减小。当侧压系数大于1时,最大剪应 力随侧压系数增加而增大;当侧压系数小于1时,最 大剪应力随侧压系数增加而减小。

# 4 结论

1)采用保角变换,应用复变函数弹性理论推导 了直墙拱形巷道围岩应力分布的解析表达式。

2)不同巷道断面高宽比、侧压系数下,直墙拱形 巷道围岩应力集中区域均主要集中在直墙底部底角 处、拱形顶板中点附近和底板中部3个位置。巷道 断面高宽比一定情况下,侧压系数大于1时,围岩边 界环向应力有相同变化趋势;采用巷道断面高宽比 小于1较有利于巷道稳定;侧压系数小于等于1时, 围岩边界环向应力也有相同变化趋势;采用巷道断 面高宽比大于1较有利于巷道稳定。

3)道断面高宽比对直墙拱形巷道沿水平线的应 力分布规律影响较小。侧压系数大于1时,巷道环 向应力均随至巷道边界距离增大而迅速增大,径向 应力均随至巷道边界距离增大而先减小后变为拉应 力然后增加,最大剪应力随侧压系数增加而增大;侧 压系数小于1时,巷道环向应力均随至巷道边界距 离增大而先增大后减小,径向应力均随至巷道边界 距离增大而增大,最大剪应力随侧压系数增加而 减小。

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